Synchronization in Physical-Layer Network Coding

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Slides in this talk based on partial content in
"Physical-layer Network Coding: Tutorial, Survey, and Beyond". A 60-page monograph.
(http://arxiv.org/abs/1105.4261)

Background

- First proposed in a challenge paper of Mobicom 06 [1]
- A subfield of Network Coding with some momentum in last two years
  - Special journal issues dedicated to the topic.
  - Citations to-date: 500+ (Google Scholar)
- Fundamental impact on *Communications*, *Information Theory*, and *Networking Research*.

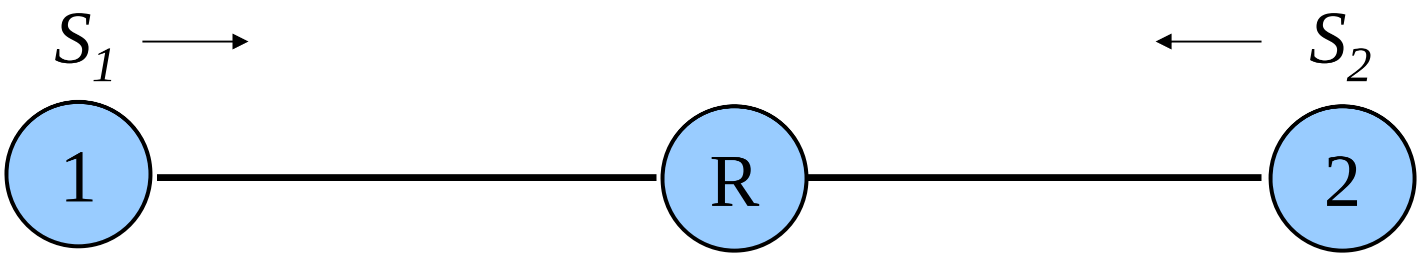
[1] S. Zhang, S. C. Liew, P. P. Lam, "Physical-Layer Network Coding," *ACM Mobicom 2006* .

Similar idea in
[2] P. Popovski and H. Yomo, "The anti-packets can increase the achievable throughput of a wireless multi-hop network," *ICC 2006*.

Outline

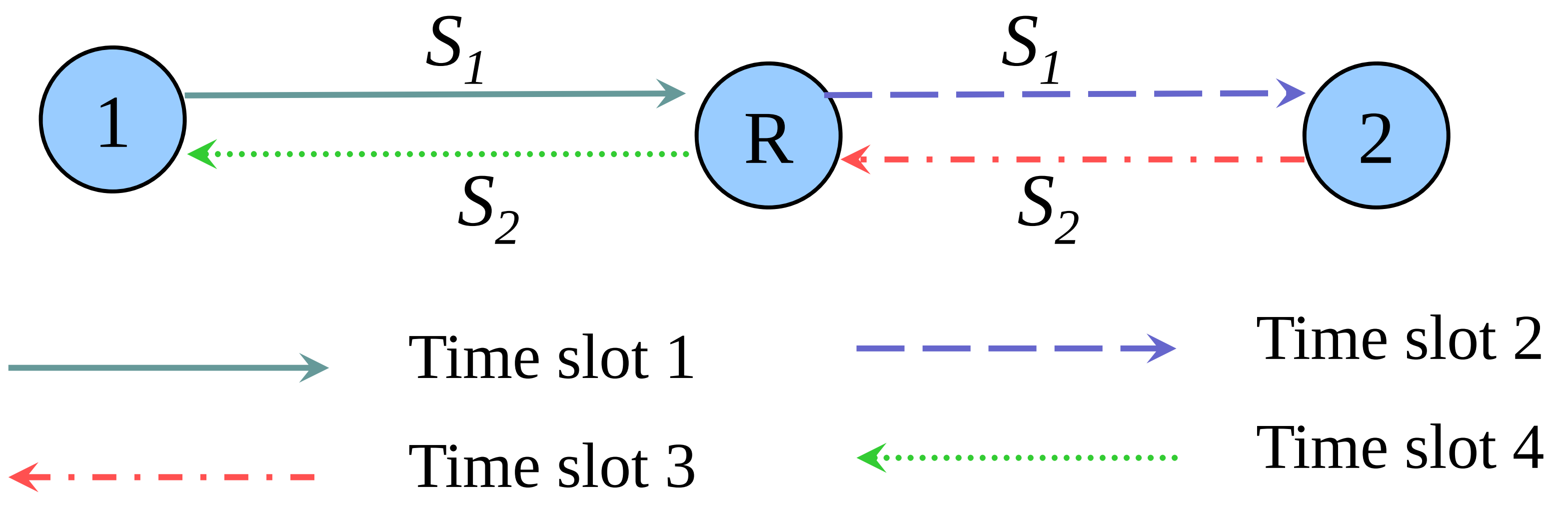
- Basic idea of PNC
- Communication-theoretic research
- Information-theoretic research
- Network-theoretic research
- Conclusion

Simplest Set-up: Two-Hop Linear Network



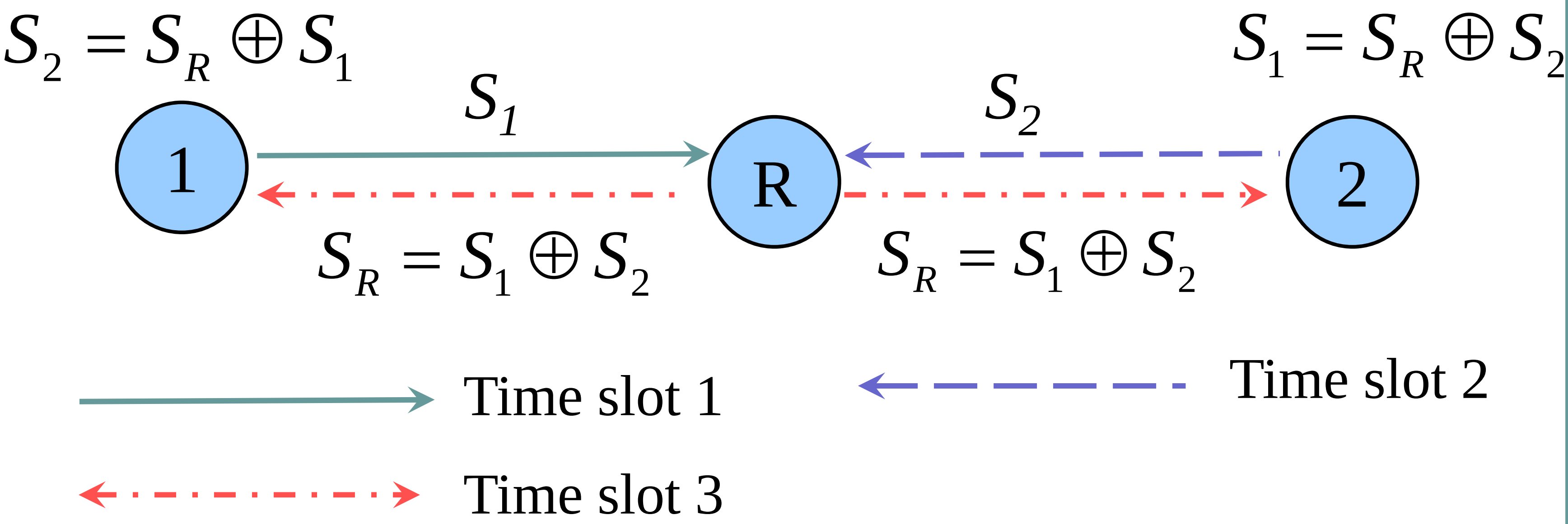
- Two-way Relay Channel (TWRC)
  - No direct channel between nodes 1 and 2.
  - Half duplex: nodes cannot transmit and receive at the same time.
  - What is the minimum number of time slots needed for nodes 1 and 2 to exchange one packet via relay node *R*?

Traditional Scheduling (TS)



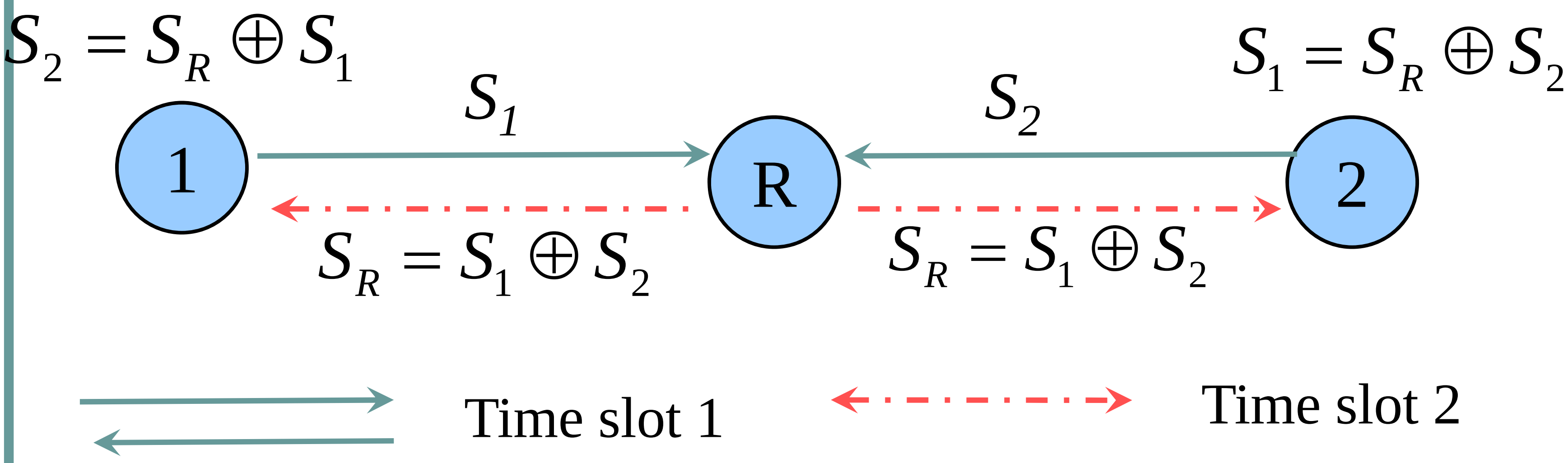
Transmissions non-overlapping in time

Straightforward Network Coding (SNC)



Node R uses one time slot to broadcast
Transmissions by nodes 1 and 2 still non-overlapping

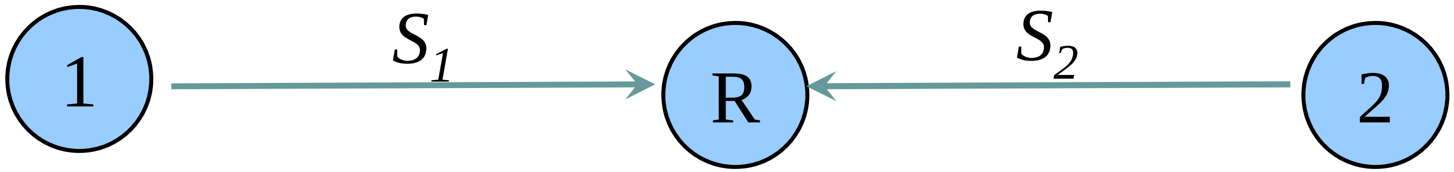
PNC



Transmissions by nodes 1 and 2 are simultaneous!

PNC Modulation Mapping (1)

- Assume BPSK modulation



$$r_R(t) =$$

$$s_1(t) \quad + \quad s_2(t)$$

$$= \quad a_1 \cos(\omega t) \quad + \quad a_2 \cos(\omega t)$$

$$= \quad (a_1 + a_2)\cos(\omega t)$$

$$\text{bit} = 1 \Leftrightarrow a_i = -1$$

$$\text{bit} = 0 \Leftrightarrow a_i = 1$$

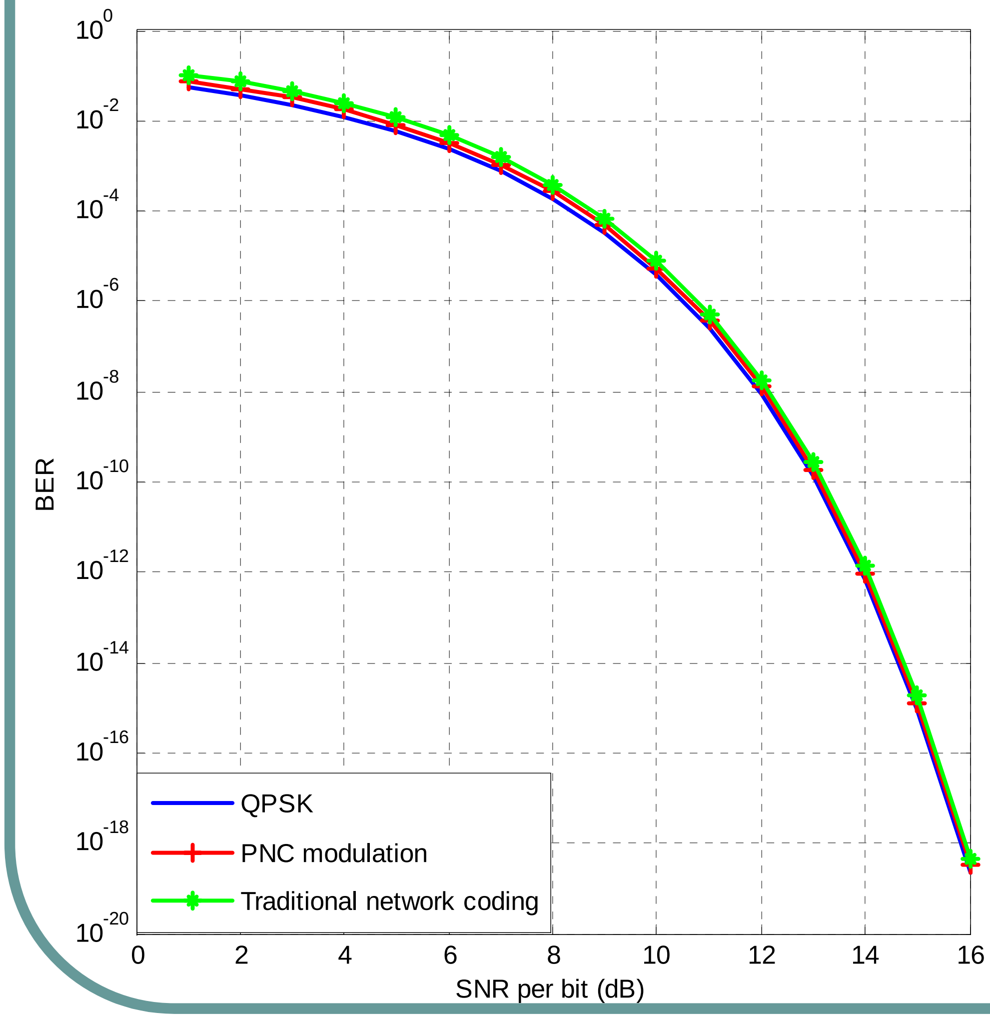
$$a_1 \oplus a_2 \equiv a_1 a_2$$

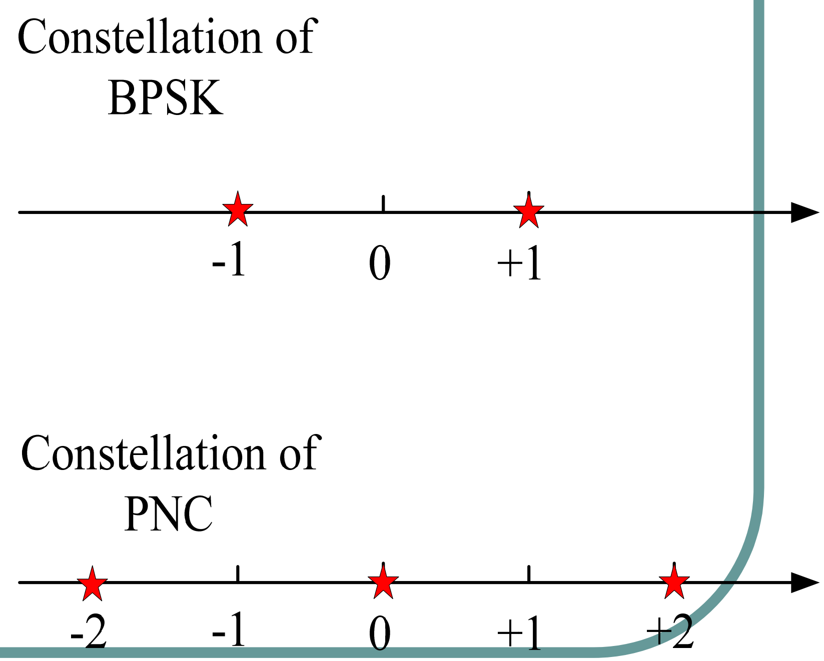
PNC Modulation Mapping (2)

a_1	a_2	$a_1 + a_2$	$a_R = a_1 a_2$
1	1	2	1
-1	1	0	-1
1	-1	0	-1
-1	-1	-2	1

Reception at relay (↑ $a_1 + a_2$) — Transmission by relay (↑ $a_R = a_1 a_2$)

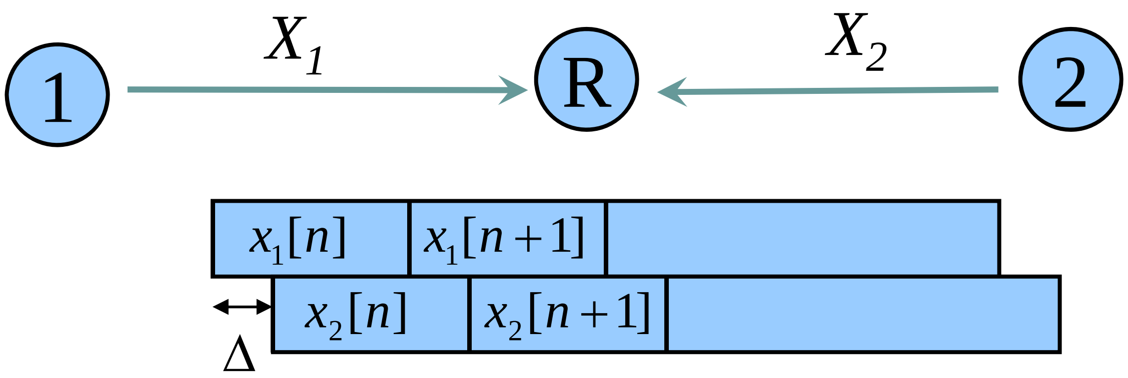
BER performance of PNC at Relay





Synchronization Penalties

(unchannel-coded case) (1)



$$y_R(t) = \sum_n x_1[n] h_{1R}\, p(t - nT) + \sum_n x_2[n] h_{2R}\, p(t - \Delta T - nT) + w_R(t)$$

$y_R(t) =$ received baseband signal

$p(t) =$ pulse shape

$\Delta =$ symbol offset

$w_R(t) =$ noise

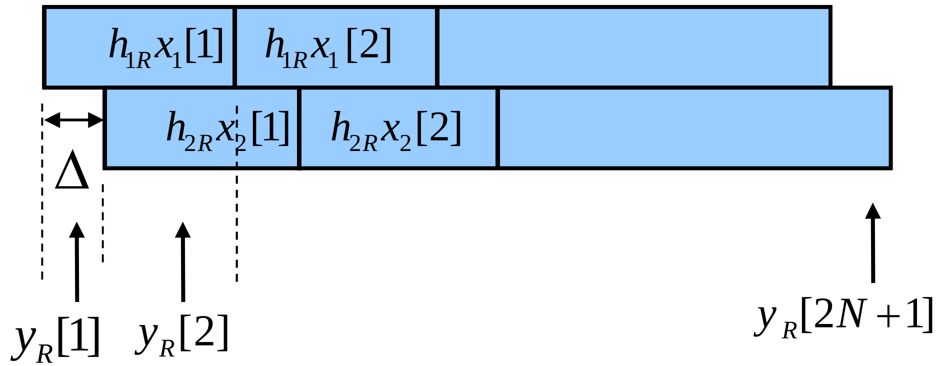
Synchronization Penalties
(unchannel-coded case) (2)

- Assumption:
  - Transmitters do not perform precoding to synchronize
  - Relay knows the channels h_{1R}, h_{2R} and symbol offset Δ
- Suboptimal results in [1] [7]
- Better results in [8]

[7] S. Zhang, S. C. Liew, P. P. Lam, "Physical-Layer Network Coding," *http://arxiv.org/abs/0704.2475*, April 2007
"On the Synchronization of Physical-Layer Network Coding," *IEEE Information Theory Workshop*, Oct 2006.
[8] L. Lu, S. C. Liew, S. Zhang, "Decoding Algorithm for Asynchronous Physical-Layer Network Coding," *IEEE ICC 2011*.

Synchronization Penalties (unchannel-coded case) (5)

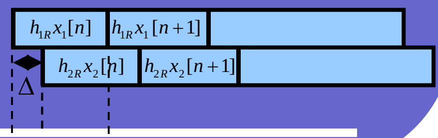
Our recent work [8] assumes oversampling and uses Belief Propagation (BP) to compute $P(x_1[n] \oplus x_2[n] \mid y_R[1], \ldots, y_R[2N+1])$ for $n = 1, \ldots, N$



Two samples per symbol

$$P(x_1[n] \oplus x_2[n] = x \mid y_R[1], \ldots, y_R[2N+1]) = \sum_{x_1[n] \oplus x_2[n] = x} P(x_1[n], x_2[n] \mid y_R[1], \ldots, y_R[2N+1])$$

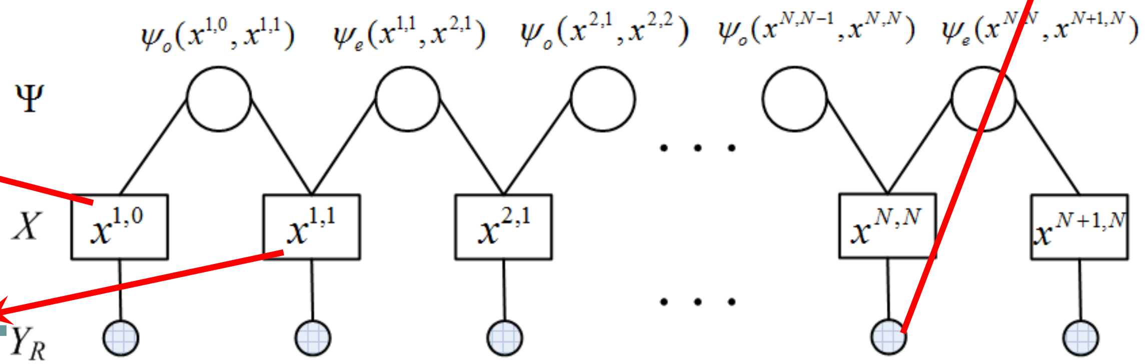
Synchronization Penalties
(unchannel-coded case) (6)



- Tanner Graph for BP decoding of $P(x_1[n], x_2[n] \mid y_R[1], \ldots, y_R[2N+1])$
  - Yields exact ML decoding because of the tree structure

$$\begin{pmatrix} \Pr\left(x_1^{(N)}=0, x_2^{(N)}=0 \mid y_R^{(N,N)}\right) \\ \Pr\left(x_1^{(N)}=0, x_2^{(N)}=1 \mid y_R^{(N,N)}\right) \\ \Pr\left(x_1^{(N)}=1, x_2^{(N)}=0 \mid y_R^{(N,N)}\right) \\ \Pr\left(x_1^{(N)}=1, x_2^{(N)}=1 \mid y_R^{(N,N)}\right) \end{pmatrix}$$

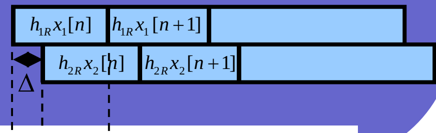
where $y_R^{(N,N)} = (1-\Delta)[h_{1R}x_1^{(N)} + h_{2R}x_2^{(N)}] + w^{(N,N)}$



$(x_1^{(1)}, \ \cdot \ )$

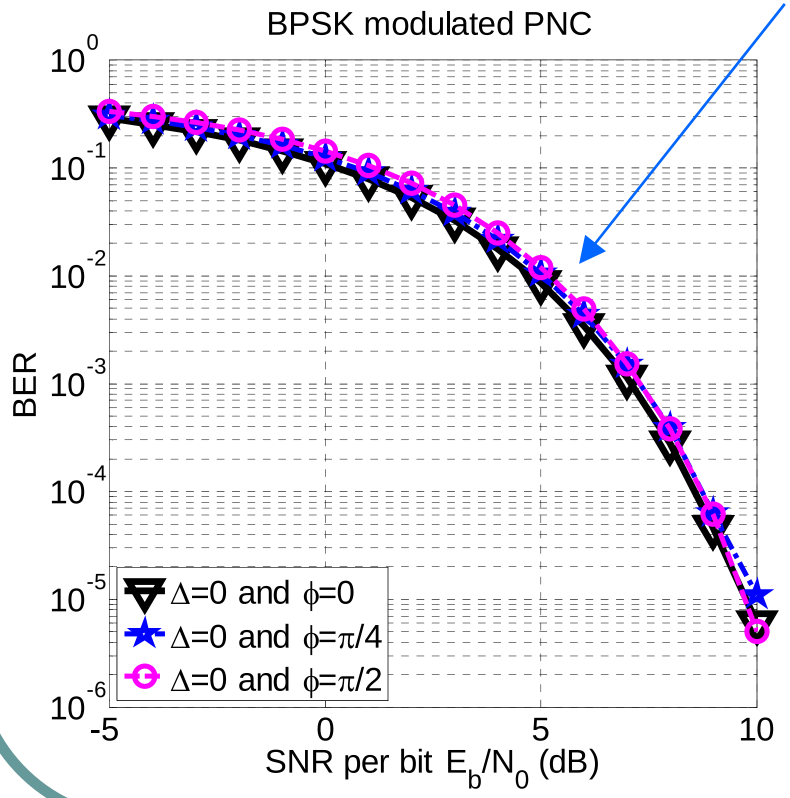
$(x_1^{(1)}, x_2^{(1)})$

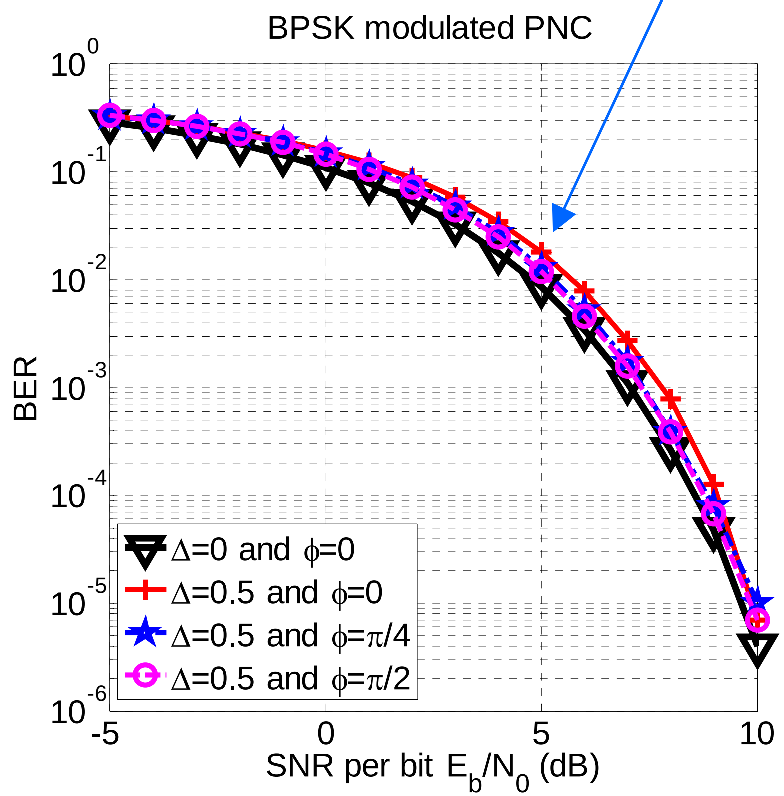
Synchronization Penalties
(Unchannel-coded case) (7)



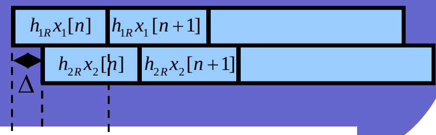
BPSK results from recent work [8]

Little penalty with phase and symbol asynchronies, thanks to BP decoding

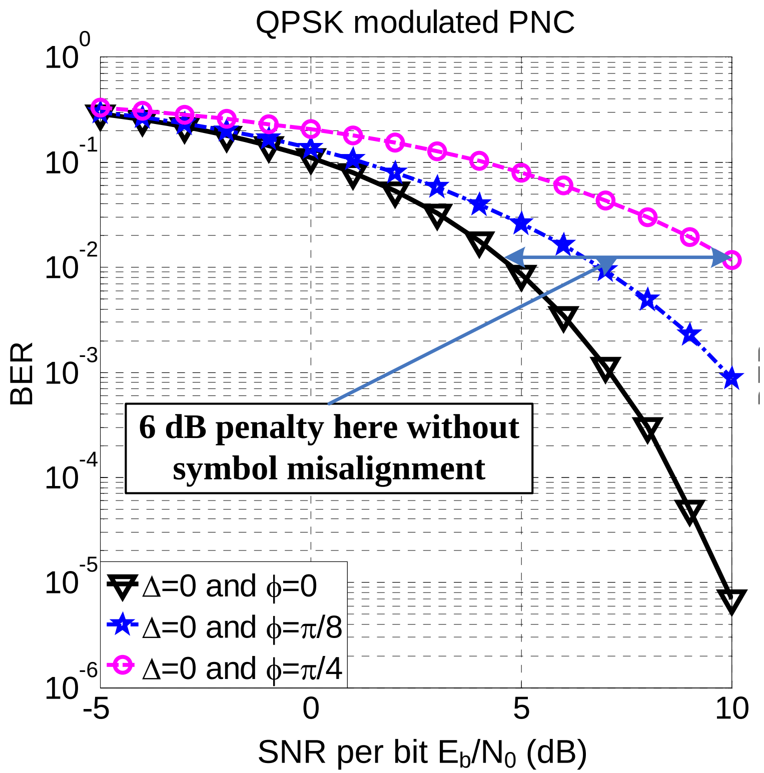




Synchronization Penalties
(Unchannel-coded case) (7)



QPSK results from recent work [8]

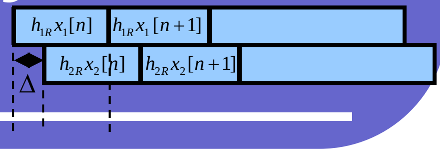


(a)

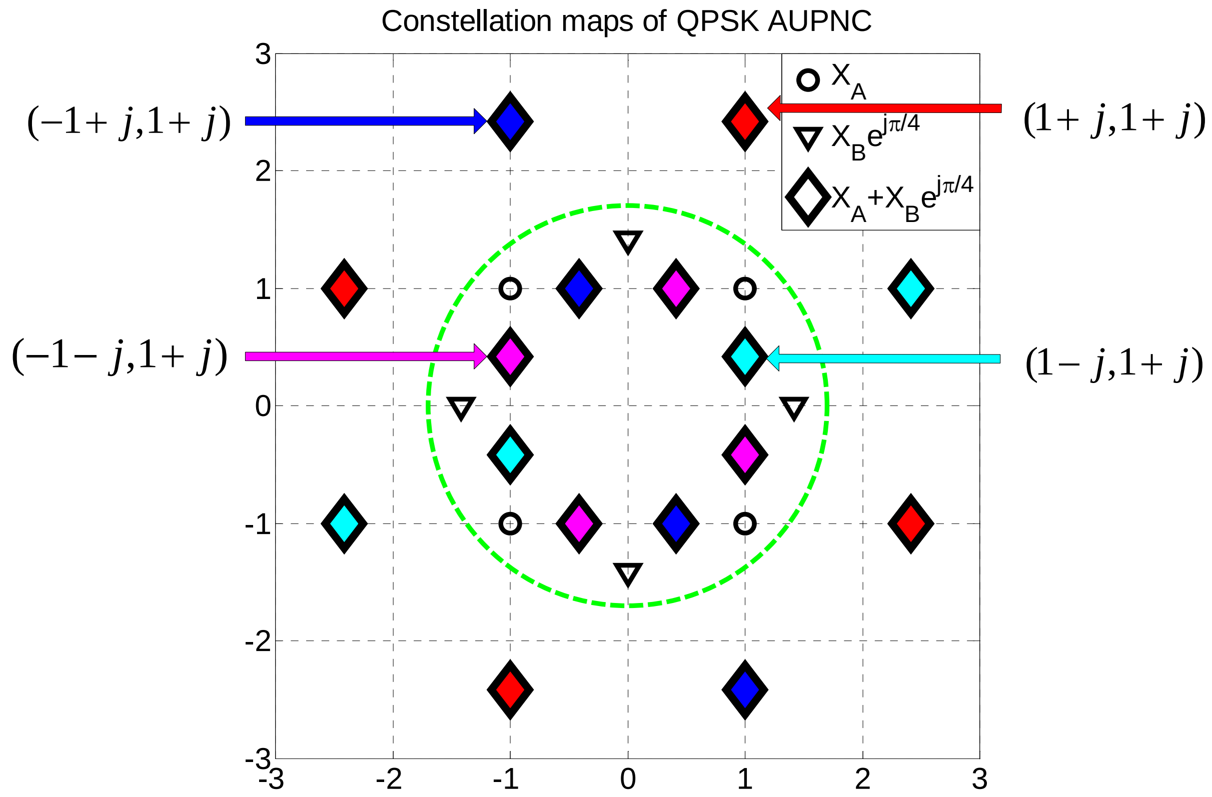
[9] Y. Hao , D. Goeckel , Z. Ding , D. Towsley , K. K. Leung, “Achievable Rates of Physical Layer Network Coding Schemes on the Exchange Channel,” *Milcom 2007*

(This paper mentions penalty of 6dB for phase asynchrony in QPSK PNC)

Certainty Propagation with Symbol Misalignment

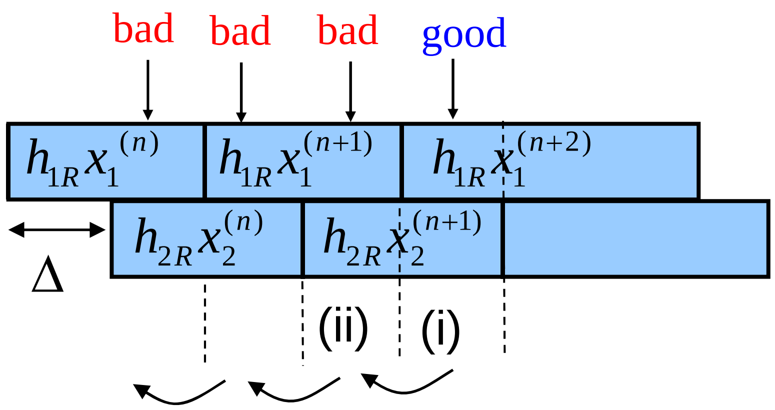


- Same-colored diamonds mapped to the same XOR PNC symbol.
- The outer 8 constellation points are separated further apart than the inner 8 constellation points.

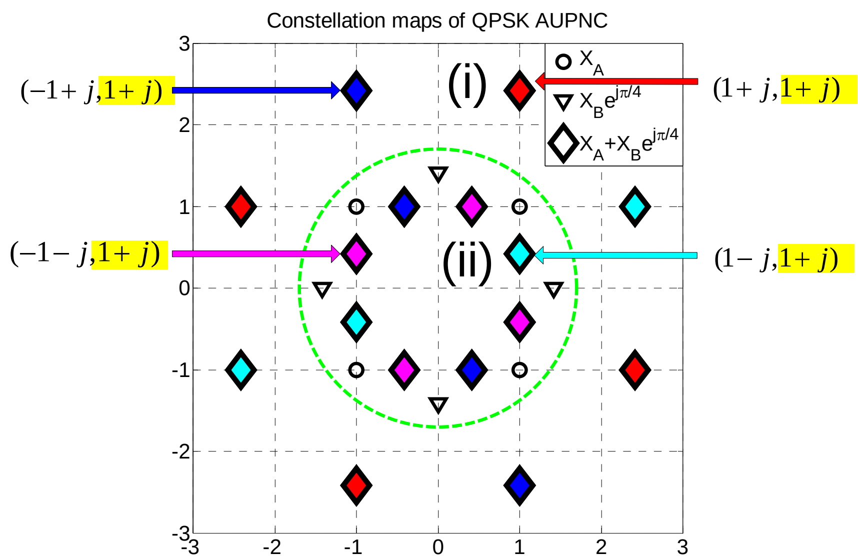


Certainty Propagation with Symbol Misalignment

- Diversity: it pays to have the information of a source symbol straddle in two constellation points.
- "Chain action": it pays even more to have symbols "chained" together in successive constellation points.



Certainty propagation
thanks to BP

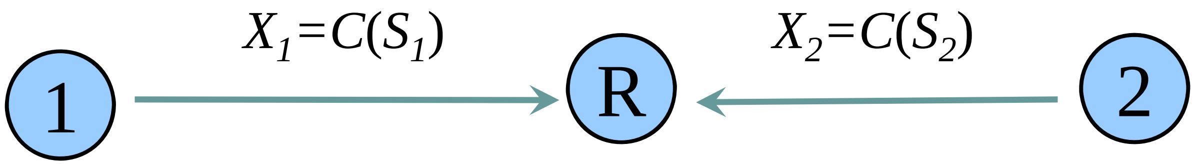


How to Incorporate Channel Coding?

- Two variants: (i) end-to-end; (ii) link-by-link
- Focus on Link-by-link

Channel-coded PNC

Multiaccess (Uplink) Phase:



Reception at relay (Δ= 0 case):

$$y_R[n] = h_{1R}x_1[n] + h_{2R}x_2[n] + w_R[n]$$

$$n = 1, \ldots, N$$

Link-by-Link Channel-coded PNC (1)

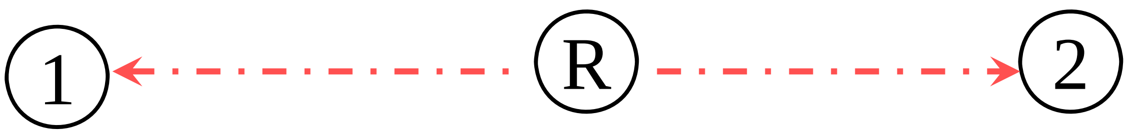
Conceptually, channel-coded PNC consists of two parts:

(i) Map $Y_R \rightarrow S_R = S_1 \oplus S_2 = (s_1[1] \oplus s_2[1], \ldots, s_1[M] \oplus s_2[M])$

(ii) Compute $X_R = C(\widehat{S}_R)$

This is similar to conventional channel coding

*This mapping involves both **C**hannel decoding and **N**etwork **C**oding. Input is N samples, output is M < N symbols when*

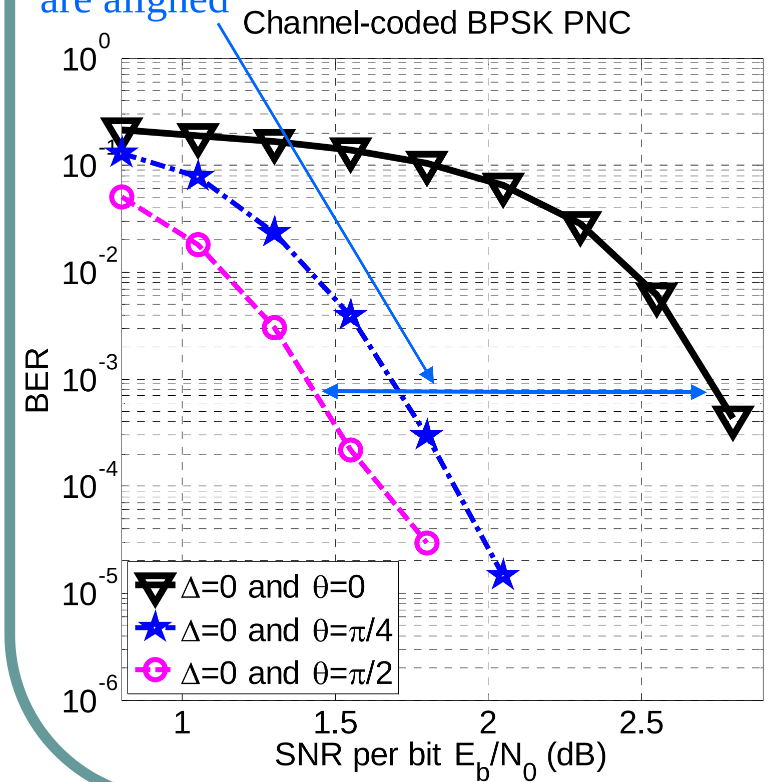


Key issue: how to do CNC(.)?

Link-by-Link Channel-coded PNC (9)

Results of $JT - CNC$ for BPSK

With channel coding, phase asynchrony results in better performance when symbols are aligned



Link-by-Link Channel-coded PNC (10)

Results of *JT-CNC* for QPSK