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Power Allocation for Two Way Relay Transmission With Analogue Network Coding

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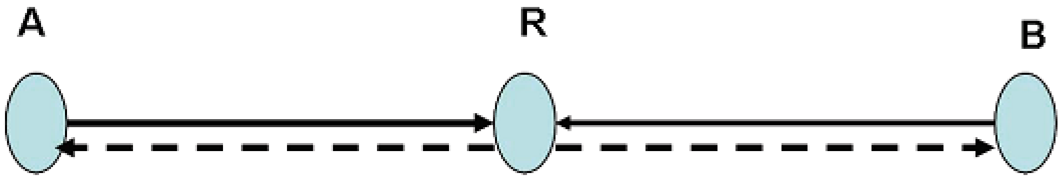
Outline

- Introduction
- Problem formulation on power allocation in tow way relay transmission with Analogue network coding
- What is the solution?
- Some numerical results
- Conclusion

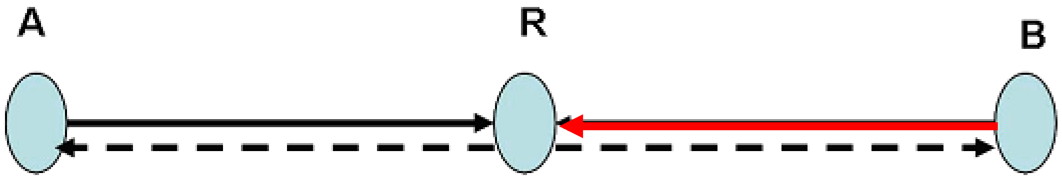
Introduction (1)

- 2000 Bob. Raymond, et al Invented “Network coding concept” in IEEE T-IT.
- 2002, Bob. Raymond, Ning developed “ Linear Network Coding” IEEE T-IT (Best Paper Award”
- Later on, Physical Layer Network coding was developed by Liew, Zhang et al. in 2006.
- One Typical Application “Two way relay network coding transmission” was investigated by many experts.

Introduction (2)



(a) Analogue Network Coding Mode



(b) Digital Network Coding Mode

Introduction (3)

Analogue Network Coding

- **Two Time Slots**
- **Keep the signal superposition form with Noise**
- **Amplitude Amplify and Forward**
- **Natural Network coding**
- **Suitable to High SNR**

Digital Network Coding

- **Three Time Slots**
- **No Signal superposition**
- **Decode, network coding and Forward**
- **Man made network coding**
- **No constraint on SNR**

Problem formulation for Power Allocation

Two different power constraints.

- **Each Node has its own average power constraint and/or power limit. (Distributed power control)**
- **All the nodes has the total average power constraint. (Centralized power control)**

Some constraints

Up link: Node A and B transmits signals to Relay

$$y_R = \sqrt{P_1} h_1 x_1 + \sqrt{P_2} h_2 x_2 + z_R$$

The maximum transmission Rate region:

$$\begin{aligned} R_1 &\leq \log(1 + P_1 |h_1|^2) \\ R_2 &\leq \log(1 + P_2 |h_2|^2) \\ R_1 + R_2 &\leq \log(1 + P_1 |h_1|^2 + P_2 |h_2|^2). \end{aligned}$$

Some Constraints

User Fairness:

Both users want to have the same amount of exchange information

$$R_1 = R_2 = \kappa \log(1 + P_1|h_1|^2) = \kappa \log(1 + P_2|h_2|^2)$$

$$2\kappa \log(1 + P_1|h_1|^2) \leq \log(1 + P_1|h_1|^2 + P_2|h_2|^2)$$

Maximize the parameter k so that the inequality above become equality

Down link: $$R_r \leq \min\{\log(1 + P_3|h_1|^2), \log(1 + P_3|h_2|^2)\}$$

Information Flow Constraint: $$t_1 R_1 = t_1 R_2 \leq t_2 R_r$$

Power Allocation Problem

Problem 1: **General Power Allocation Problem**

$$\max_{t1,t_2,P_1,P_2,P_3} \{R_{AWGN} = \max\{t_1\kappa\log(1+P_1|h_1|^2) + t_1\kappa\log(1+P_2|h_2|^2)\}\}$$

subject to

$$\begin{aligned}
t_1 + t_2 &= 1, t_1 > 0, t_2 > 0 \\
t_1\kappa\log(1+P_1|h_1|^2) &= t_1\kappa\log(1+P_2|h_2|^2) \\
\kappa\{\log(1+P_1|h_1|^2) + \log(1+P_2|h_2|^2)\} &\leq \log(1+P_1|h_1|^2 + P_2|h_2|^2) \\
t_1\kappa\log(1+P_1|h_1|^2) &= t_2\min\{\log(1+P_3|h_1|^2), \log(1+P_3|h_2|^2)\} \\
t_1P_1 + t_1P_2 + t_2P_3 &\leq E_P
\end{aligned}$$

Another Form Problem

Problem 1A:

$$\max_{t1,t_2,P_1,P_2,P_3} \{R_{AWGN} = \max\{t_1 \log(1+2P_1|h_1|^2)\}$$

$$\begin{aligned}
t_1 + t_2 &= 1, t_1 > 0, t_2 > 0 \\
P_1 &= \beta P_2 \\
\frac{t_1}{2}\log(1+2P_1|h_1|^2) &= t_2 \min\{\log(1+P_3|h_1|^2), \log(1+P_3|h_2|^2)\} \\
t_1(1+\beta)P_2 + t_2 P_3 &= E_P
\end{aligned}$$

where

$$: \beta = \frac{|h_2|^2}{|h_1|^2}.$$

What is the solution?

Answer: **No Explicit Solution**

How to find a near optimal solution with explicit expression?

One Special Case

Problem 2

$$\max_{P_1,P_2,P_3} \{R_{AWGN} = \max\{\frac{2}{3}\log(1+2P_1|h_1|^2)\}$$

subject to

$$\begin{aligned} P_1 &= \beta P_2 \\ \log(1+2P_1|h_1|^2) &= \min\{\log(1+P_3|h_1|^2), \log(1+P_3|h_2|^2)\} \\ \frac{2}{3}(1+\beta)P_2 + \frac{P_3}{3} &= E_P \end{aligned}$$

where $t_1 = 2/3$ and $t_2 = 1/3$

Solution to the Special Case

Theorem 1

The optimal power allocation to **Problem 2** is given by

$$P_1 : P_2 : P_3 = \beta : 1 : 2,$$

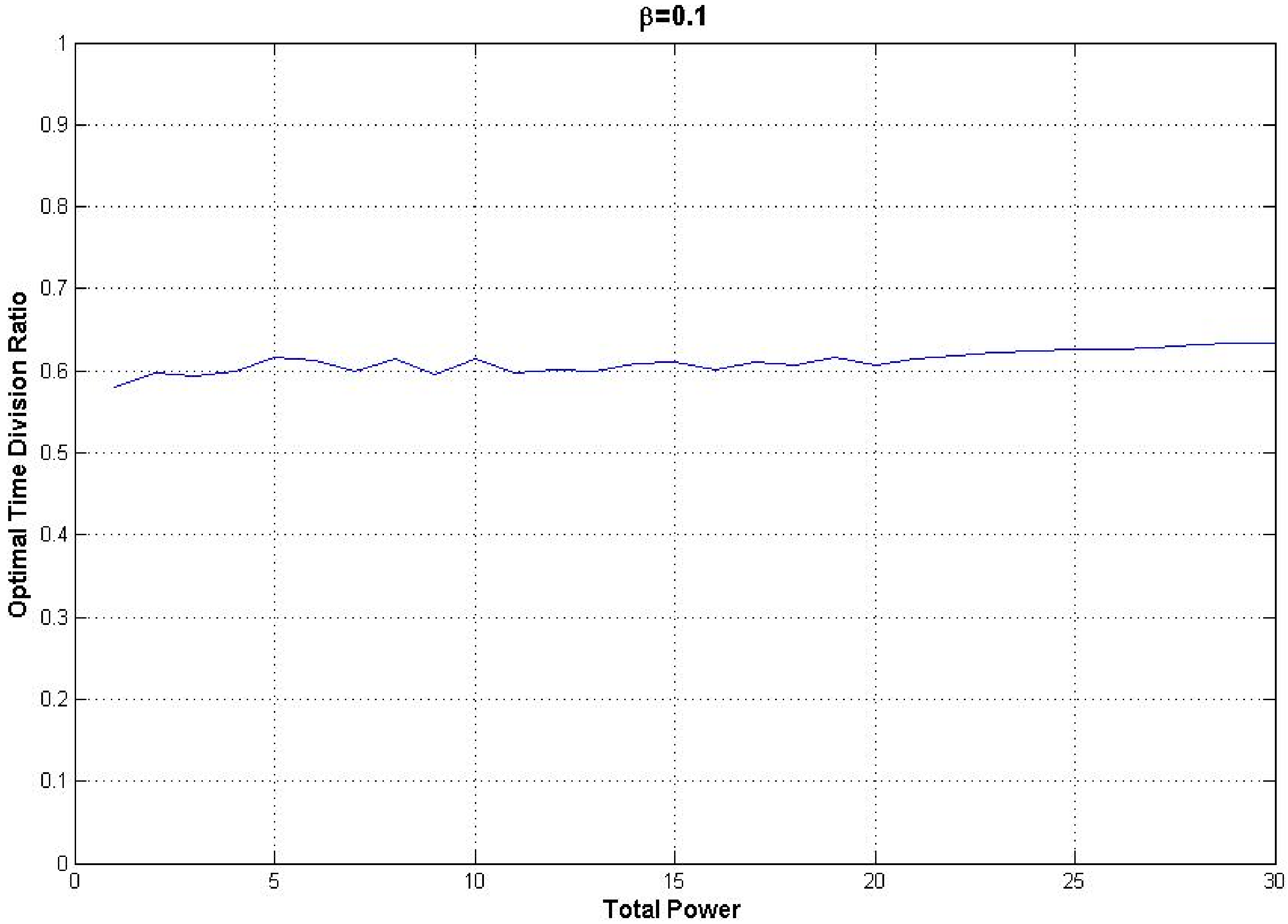
and the optimal energy allocation ratio is given by

$$t_1 P_1 : t_1 P_2 : t_2 P3 = \beta : 1 : 1$$

for $0 < \beta < 1$ and the corresponding maximum exchange information rate is

$$R_{AWGN} = \frac{2}{3}\log(1 + \frac{3\beta E_P}{4 + 2\beta}|h_1|^2) = \frac{2}{3}\log(1 + \frac{3|h_1|^2|h_2|^2}{2|h_1|^2 + |h_2|^2}E_P).$$

Numerical Results



On digital Network Coding

Up link:

$$C_{AR} = \log(1 + P_1|h_1|^2)$$
$$C_{BR} = \log(1 + P_2|h_2|^2)$$

Down link:

$$C_{RA} = \log(1 + P_3|h_1|^2)$$
$$C_{RB} = \log(1 + P_3|h_2|^2)$$

Power Allocation for DNC

Problem 3:

$$\max_{t1,t_2,t_3,P_1,P_2,P_3} \{R_{AWGN} = \max\{t_1 \log(1+P_1|h_1|^2) + t_2 \log(1+P_2|h_2|^2)\}\}$$

subject to

$$\begin{aligned}
t_1 + t_2 + t_3 &= 1, t_1 > 0, t_2 > 0, t_3 > 0 \\
t_1 \log(1+P_1|h_1|^2) &= t_2 \log(1+P_2|h_2|^2) \\
t_1 \log(1+P_1|h_1|^2) &= t_3 \min\{\log(1+P_3|h_1|^2), \log(1+P_3|h_2|^2)\} \\
t_1P_1 + t_2P_2 + t_3P_3 &\leq E_P
\end{aligned}$$

Another Form on DNC

Problem 3A

$$\max_{t,t_3,P_1,P_2,P_3} \{R_{AWGN} = \max\{t\log(1+P_1|h_1|^2)\}\}$$

subject to

$$\begin{aligned} t + t_3 &= 1, t > 0, t_3 > 0 \\ P_1 &= \beta P_2 \\ \frac{t}{2}\log(1+P_1|h_1|^2) &= t_3 \min\{\log(1+P_3|h_1|^2), \log(1+P_3|h_2|^2)\} \\ \frac{t}{2}(1+\beta)P_2 + t_3 P_3 &= E_P \end{aligned}$$

with $\beta = |h_2|^2/|h_1|^2$, $t_1 = t_2 = \frac{t}{2}$.

Special Case:

Problem 4

$$\max_{P_1,P_2,P_3} \{R_{AWGN} = \max\{\frac{2}{3}\log(1+P_1|h_1|^2)\}\}$$

subject to

$$\begin{aligned} P_1 &= \beta P_2 \\ \log(1+P_1|h_1|^2) &= \min\{\log(1+P_3|h_1|^2), \log(1+P_3|h_2|^2)\} \\ \frac{1}{3}(1+\beta)P_2 + \frac{1}{3}P_3 &= E_P \end{aligned}$$

Solution to Special Case of DNC

Theorem 2

The optimal power allocation to **Problem 4** is given by

$$P_1 : P_2 : P_3 = \beta : 1 : 1,$$

and the optimal energy allocation ratio is given by

$$t_1 P_1 : t_1 P_2 : t_2 P3 = \beta : 1 : 1$$

for $0 < \beta < 1$ and the corresponding maximum exchange information rate is

$$R_{AWGN} = \frac{2}{3}\log(1 + \frac{3\beta E_P}{2+\beta}|h_1|^2) = \frac{2}{3}\log(1 + \frac{3|h_1|^2|h_2|^2}{2|h_1|^2 + |h_2|^2}E_P).$$

Observations and Conclusion

- Digital network coding with three equal time slots division scheduling can achieve the maximum exchange information rate of multiple access mode if proper power allocation is employed.

- Digital network coding is near optimal when user fairness is taken into account.

- Set up a bridge between Analogue Network Coding and Digital Network Coding

Thanks for the cooperation with Raymond Yeung on this topic

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Thanks for your attention!

Question?